Laws of Collision / demonstration track with Cobra4

Related topics

Conservation of momentum, conservation of energy, linear motion, velocity, elastic loss, elastic collision, inelastic collision.

Principle

The velocities of two carts, moving on a demonstration track, are measured before and after collision, for both elastic and inelastic collision.

Equipment

1	Cobra4 Wireless Manager	12600-00
1	Cobra4 Wireless-Link	12601-00
1	Cobra4 Sensor-Unit Timer/Counter	12651-00
1	Cobra4 adapter for Sensor-Unit Timer/Counter	12651-01
1	Demonstration Track, Aluminium, Length: 1.5 m	11305-00
2	Light barrier, compact	11207-20
2	Needle with plug	11202-06
1	Fork with plug	11202-08
1	Rubber bands f.fork w/plug,10 pcs.	11202-09
1	Plate with plug	11202-10
1	Magnet w.plug f.starter system	11202-14
1	Plasticine, 10 sticks	03935-03
10	Slotted weight, 10 g, black	02205-01
6	Slotted weight, 50 g, black	02206-01
1	Portable Balance, OHAUS CS2000 - AC adapter included	48917-93
2	Cart, low friction sapphire bearings	11306-00
1	Starter system for demonstration track	11309-00
2	Weight for low friction cart, 400 g	11306-10
2	Shutter plate for low friction cart,	11308-00
2	Holder for light barrier	11307-00
1	End holder for demonstration track	11305-12
2	Connecting cord, 32 A, 750 mm, red	07362-01
2	Connecting cord, 32 A, 750 mm, blue	07362-04

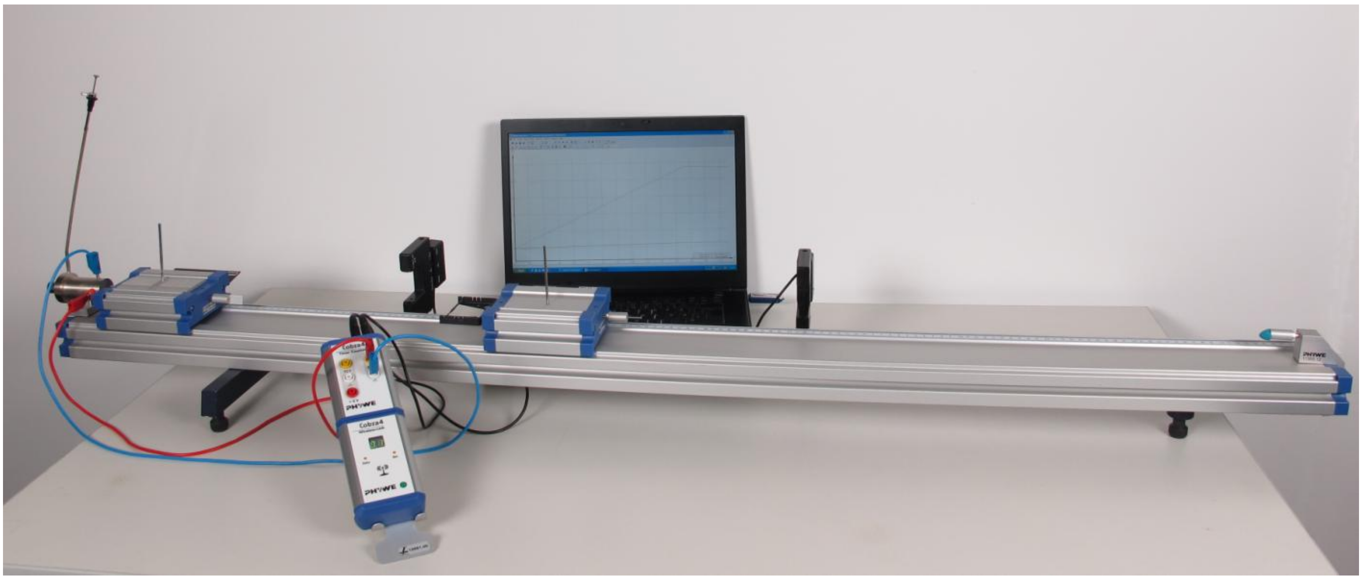

Fig. 1: P2130560.

Tasks

1. Elastic collision

A cart collides with a second resting cart at a constant velocity. A measurement series is conducted by varying the mass of the resting cart: The corresponding velocities of the first cart before the collision and the velocities of both carts after it are to be measured. Plot the following parameters as functions of the mass ratio of the carts:

1.1 The impulses of the two carts as well as their sum after the collision. For comparison the mean value of the impulses of the first cart is entered as a horizontal line in the graph.

1.2 Their energies, in a manner analogous to Task 1.1.

1.3 In accordance with the mean value of the measured impulse of the first cart before the collision, the theoretical values of the impulses for the two carts are entered for a range of mass ratios from 0 to 3. For purposes of comparison the measuring points (see 1.1) are plotted in the graph.

1.4 In accordance with the mean value of the measured energy of the first cart before the collision, the theoretical values of the energy after the collision are plotted analogously to Task 1.3. In the process, the measured values are compared with the theoretical curves.

2. Inelastic collision

A cart collides with a constant velocity with a second resting cart. A measurement series with different masses of the resting cart is performed: the velocities of the first cart before the collision and those of both carts, which have equal velocities, after it are to be measured.

2.1 The impulse values are plotted as in Task 1.1.

2.2 The energy values are plotted as in Task 1.2.

The theoretical and measured impulse values are compared as in Task 1.3.

As in Task 1.4, the theoretical and measured energy values are compared. In order to clearly illustrate the energy loss and its dependence on the mass ratios, the theoretical functions of the total energy of both carts and the energy loss after the collision are plotted.

Set up and procedure

The experimental set-up is performed as shown in Fig. 1.

- Connect the light barriers with the adapter to the Sensor-Unit Timer/Counter. Start the PC and Windows.
- Plug Cobra4 Wireless Manager in the USB-port of the PC.
- Start the measure software package on the PC.
- Switch on the Cobra4 Wireless-Link with plugged on Timer-Counter Sensor-Unit. The sensor is now automatically recognized and is allocated the ID number 01 which is displayed in the Cobra4 Wire-less-Link display. Communication between Cobra4 Wireless Manager and Cobra4 Wire-less-Link is shown via the *Data* LED.
- Use the compact balance to measure the mass of the cart *m2* with attached rod for holding additional weights, holding magnet and needle with plug.
- Load the experiment in measure (Experiment > Open experiment). All pre-settings which are necessary for the recording of measured values are now started.
- Start the recording of measured values in measure ●.
- The starting device serves to start the cart; three defined and reproducible initial energies can be selected with the various latch positions. It is recommended that the second position is used for all measurements.

The momentum is determined by measuring the velocity of the cart. For this purpose, the time during which the screen fitted on the cart impinges on the light barrier is used, in accordance with:

$$v = \frac{\Delta s}{\Delta t}$$

(Δs = length of screen, Δt = shading time).

Theory and evaluation

In the elastic collision of two bodies having masses m_1 and m_2, kinetic energy and momentum are conserved:

$$\frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} = \frac{\vec{p}_1^{\,\prime 2}}{2m_1} + \frac{\vec{p}_2^{\,\prime 2}}{2m_2}$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1^{\,\prime} + \vec{p}_2^{\,\prime},$$

where $\vec{p}_1$, $\vec{p}_2$ are the moments before the collision and $\vec{p}_1^{\,\prime}$, $\vec{p}_2^{\,\prime}$ those after the collision.

Due to the unidimensional sequence of movement, we will dispense with the vectorial notation. For a central elastic with $p_2 = 0$:

$$p_1' = \frac{m_1 - m_2}{m_1 + m_2} \cdot p_1 = -\frac{1 - \frac{m_1}{m_2}}{1 + \frac{m_1}{m_2}} \cdot p_1$$

$$p_2' = \frac{2m_2}{m_1 + m_2} \cdot p_1 = \frac{2}{1 + \frac{m_1}{m_2}} \cdot p_1$$

From the contribution of the impulse p, the energies E can be calculated according to $E = p_2/2m$:

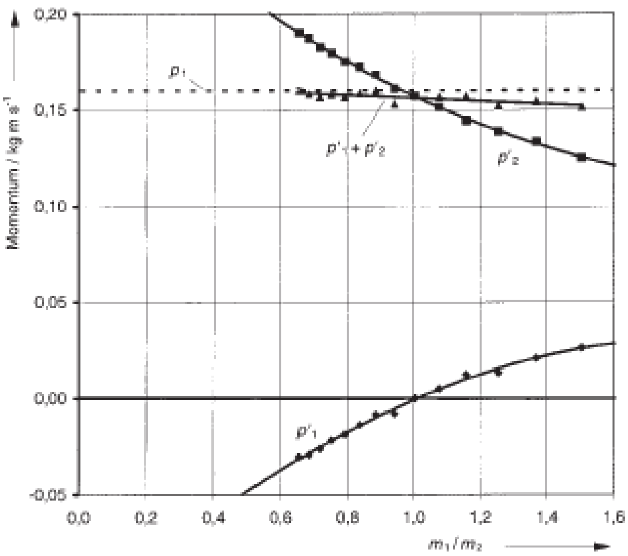

Fig. 2: Elastic collision: moment after the collision as functions of the mass ratio of the carts.

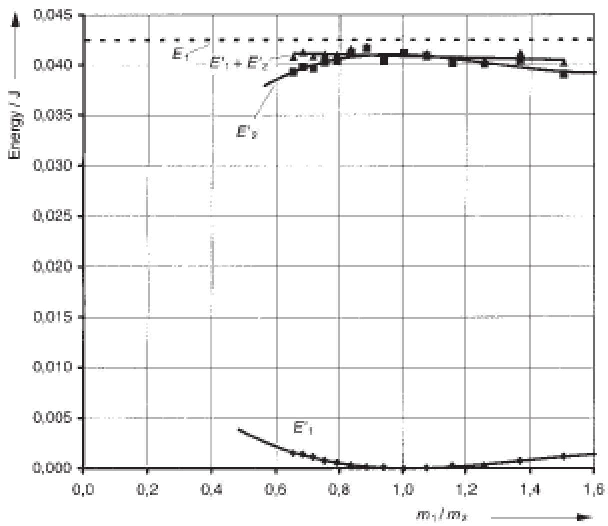

Fig. 3: Elastic collision: energy after the collision as functions of the mass ratio of the carts.

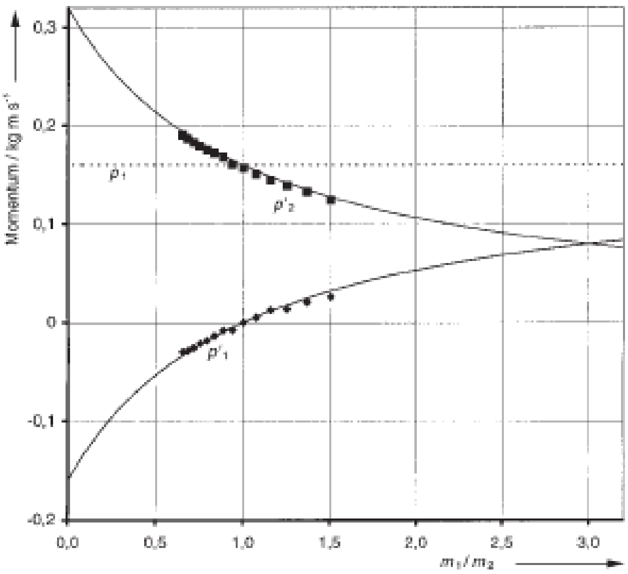

Fig. 4: Elastic collision: calculated momenta after the collision as functions of the mass ratio of the carts.

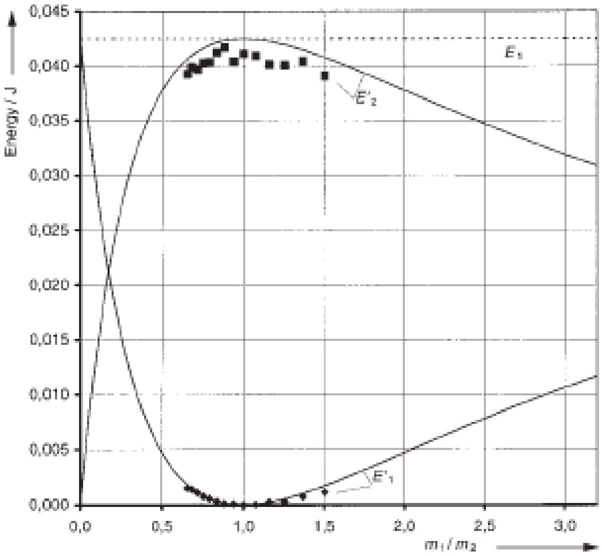

Fig. 5: Elastic collision: calculated energies after the collision as functions of the mass ratio of the carts.

$$E_1' = -\left(\frac{1-\frac{m_1}{m_2}}{1+\frac{m_1}{m_2}}\right)^2 \cdot E_1$$

$$E_1' = -\frac{4}{\left(1+\frac{m_1}{m_2}\right)^2} \cdot \frac{m_1}{m_2} \cdot E_1$$

Fig. 2 and Fig. 3 show the results for a sample measurement. In particular, one can see that the total impulse and the total energy before and after the collision, except for a slight loss, are equal. The comparison of the measured values with the theoretical values according to the formulas given above can be seen in Fig. 4 and Fig. 5.

In an inelastic collision, only the momentum is conserved. In addition, the velocities after the collision are equal:

$$p_1' = \frac{m_1}{m_2} \cdot p_2'$$

Therefore,

$$p_1' = \frac{1}{1+\frac{m_2}{m_1}} \cdot p_1$$

$$p_2' = \frac{1}{1+\frac{m_1}{m_2}} \cdot p_1$$

The following is obtained for the energies of the two carts after the collision:

$$E_1' = \frac{1}{\left(1+\frac{m_2}{m_1}\right)^2} \cdot E_1$$

$$E_2' = \frac{1}{\left(1+\frac{m_1}{m_2}\right)^2} \cdot \frac{m_1}{m_2} \cdot E_1$$

The evaluation of a sample measurement (Fig. 6 and Fig. 7) shows that also for an inelastic collision, the total impulse is conserved; whereas, depending on m_1/m_2, a substantial energy loss occurs.

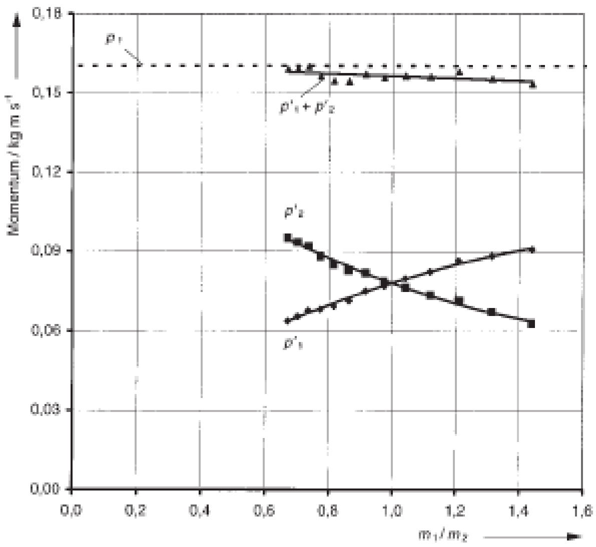

Fig. 6: Inelastic collision: momenta after the collision as functions of the mass ratio of the carts.

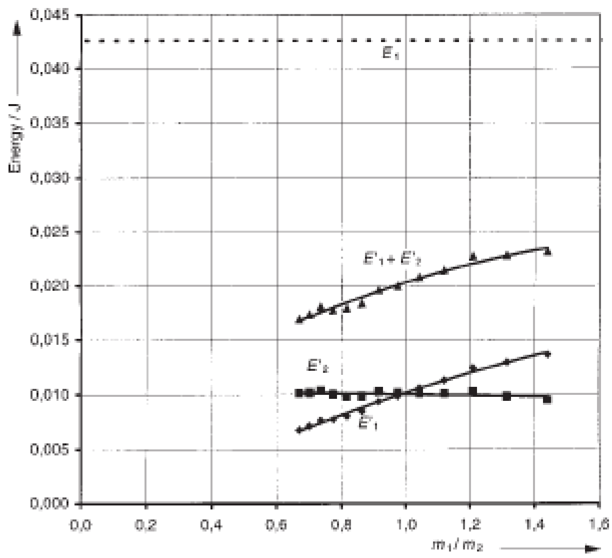

Fig. 7: Inelastic collision: energy after the collision as functions of the mass ratio of the carts.

The theoretical curves are compared with the measured values in Fig. 8 and Fig. 9. In Fig. 9, the energy loss is additionally plotted [energy loss = $E_1 - (E_1' + E_2')$]. One sees that for a mass ratio of 1, the kinetic energy is reduced by exactly 50%.

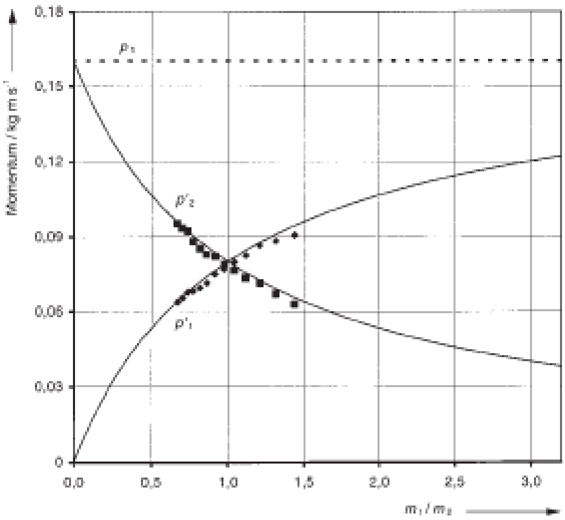

Fig. 8: Inelastic collision: calculated moment after the collision as functions of the mass ratio of the carts.

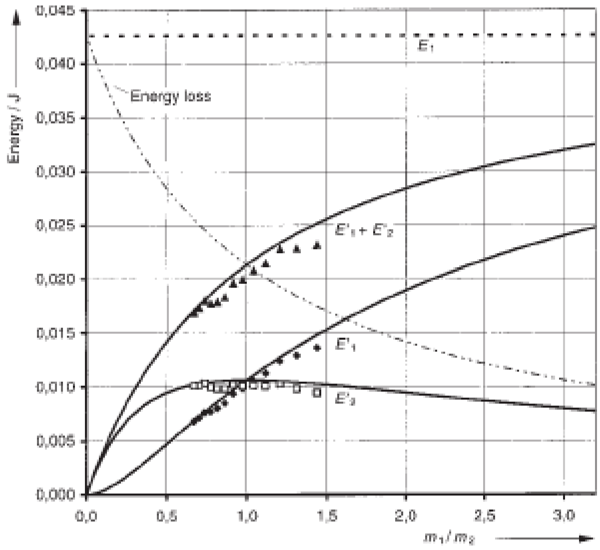

Fig. 9: Inelastic collision: calculated energies after the collision and energy loss as functions of the mass ratio of the carts.